Rauli Susmel
Dept. of Finance
Univ. of Houston

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Chapter 8 - Theories of FX Determination – Part 1

In Chapter 4, we briefly mentioned two theories of exchange rate determination: The Balance of Trade (BT) Approach and the Monetary Approach (MA)

Under the BT Approach, net trade flows (X-M) are the main determinants of S_t. According to this approach, we expect an increase (decrease) in the TB to depreciate (appreciate) the FC. That is,

$$e_{f,t} = \frac{S_{t+T} - S_t}{S_t} = f(TB_t), \qquad \text{where } f' < 0.$$

Under the MA, S_t is determined by the relative money demand and money supply between the two currencies:

$$S_t = f(L_{d,T} / L_{f,T} , M_{Sd,T} / M_{Sf,T},,...), \qquad \text{where } f_1 < 0 \;\&\; f_2 > 0.$$

In this chapter, we develop more theories to explain S_t. The emphasis will be on arbitrage, actually *pseudo-arbitrage*, theories, focusing on equilibrium in only one market. That is, we will rely on *partial equilibrium* stories to explain S_t.

Our goal is to find an explicit functional form for S_t, say $S_t = \alpha + \beta X_t$, where X_t. is a variable or set of variables determined by a theory. Different theories will have different X_t and or different *f*(.).

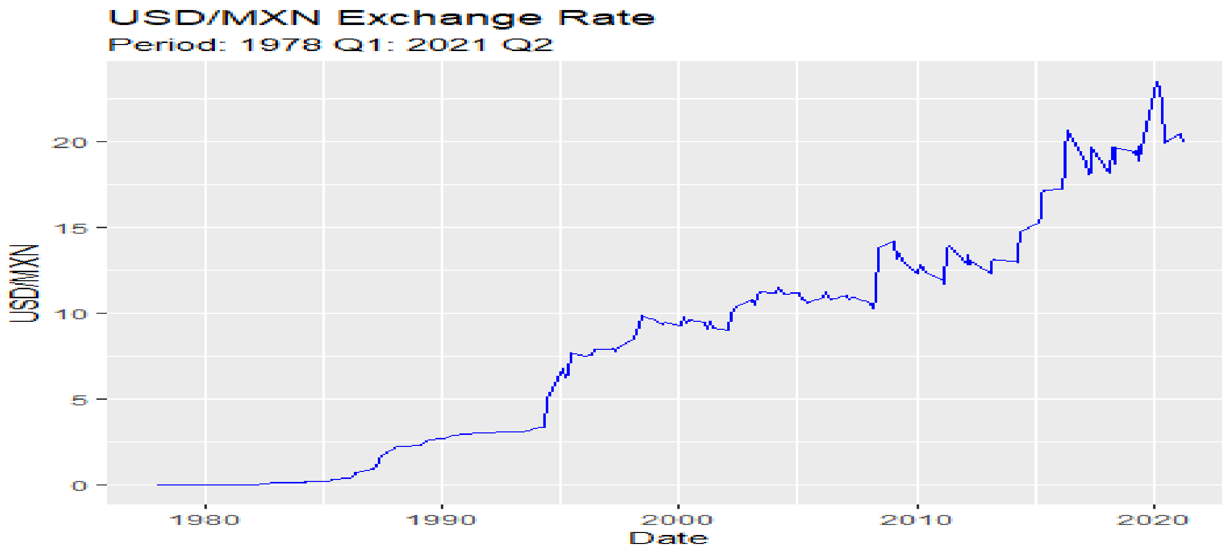
Eventually, we would like to have a precise mathematical formula to forecast S_{t+T}.

Q: How do we know the formula of S_t is any good?

• Testing a Theory

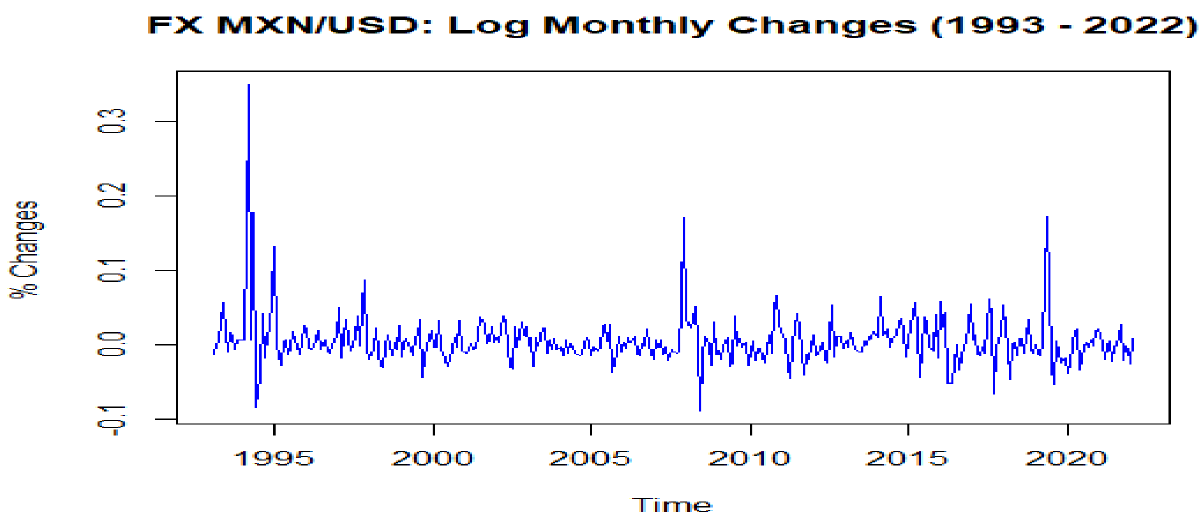
We will judge a theory by how well it explains the behavior of the observed S_t. For example, a good theory should match the observed behavior of the MXN/USD exchange rate for the 1987-2022 period, as shown in Figure 8.1.

Figure 8.1: Behavior of the MXN/USD (1987-2022)



Like many macroeconomic series, exchange rates have a trend, see Figure 8.1 above –in statistics, these trends in macroeconomic series are called *stochastic trends.* It is better to work with changes, not levels. As can be seen in Figure 8.2, the trend is gone after calculating changes in S_t.

Figure 8.2: Behavior of the Changes in MXN/USD (1987-2022)



Now, the trend, which in many cases is easy to explain, is gone. Our goal will be to explain $e_{f,t}$, the percentage change in S_t.

Goal: S_t = f(i_d, i_f,I_d, I_f,…). But, it'll be easier to explain $e_{f,t} = \frac{S_{t+T} - S_t}{S_t}$ = f(i_d, i_f,I_d, I_f,…).

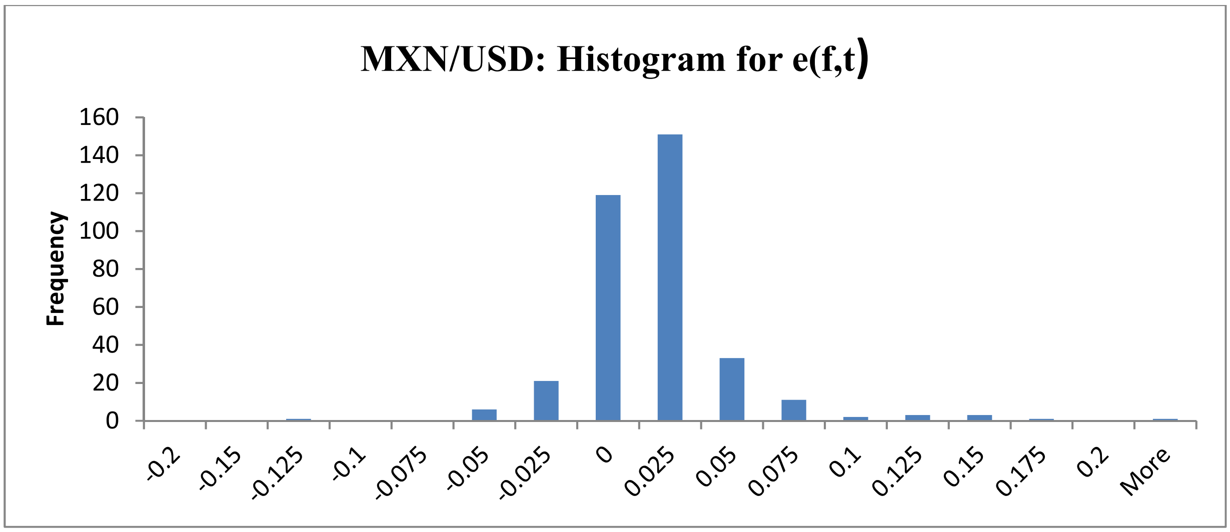
Once we get $e_{f,t}$, we get S_t => $S_t = S_{t-1} * (1 + e_{f,t})$

The S_t that we'll obtain will be an *equilibrium value*. That is, the S_t we will be calculated using a model that assumes some kind of equilibrium in the FX market.

Q: How are we going to test our equilibrium values?
A: We would like our theory to match the data, say the mean and standard deviation of S_t.

Figure 8.3 plots the distribution of $e_{f,t}$(MXN/USD); calculated from monthly data during 1987-2017. Below Figure 8.3, we show the descriptive statistics for the distribution of $e_{f,t}$(MXN/USD).

Figure 8.3: Distribution of the Changes in MXN/USD (1987-2017)



Descriptive Stats:

$e_{f,t}$(MXN/USD)	
Mean	0.006732
Standard Error	0.002024
Median	0.00306
Mode	0
Standard Deviation	0.037973
Sample Variance	0.001442
Kurtosis	55.83344
Skewness	5.217048
Range	0.58129
Minimum	-0.12822
Maximum	0.453066
Sum	2.369586
Count	352

The usual (average, expected) monthly percentage change represents a 0.67% appreciation of the USD against the MXN (annualized change: 8.38%). The standard deviation is 3.80%. The mean, standard deviation, skewness and kurtosis are called *unconditional moments*. Theories also produce *conditional moments* -i.e., conditional on the theory/models. In general, we associate matching unconditional moments with long-run features of a model; while we associate matching conditional moments with short-run features of a model.

• Review from Chapter 7
Effect of arbitrage on FX Markets:

Local arbitrage → sets consistent rates across banks
Triangular arbitrage → sets cross rates
Covered arbitrage → sets a relation between $F_{t,T}, S_t, i_d\ i_f$ (IRPT)

$$\Rightarrow F_{t,T} = S_t * \frac{\left(1+i_d * \frac{T}{360}\right)}{\left(1+i_f * \frac{T}{360}\right)}$$

This Lecture
In this class, we will study the effect of "*arbitrage*" in goods (PPP) and financial flows (IFE) on FX Markets. We will generate explicit models for S_t. (Always keep in mind that all models are simplifications of the real world.)

8.1 Purchasing Power Parity (PPP)

PPP is based on the law of one price (LOOP): same goods once denominated in a common currency should have the same price. If they are not, then pseudo-arbitrage is possible.

Example: LOOP for Oil.
$P_{oil\text{-}USA}$ = **USD 40**.
$P_{oil\text{-}SWIT}$ = **CHF 80**.
LOOP: $P_{oil\text{-}SWIT}\ S_t^{LOOP} = P_{oil\text{-}USA}$
$\Rightarrow S_t^{LOOP} = P_{oil\text{-}USA}/P_{oil\text{-}SWIT}$ = **USD 40**/**CHF 80** = **0.50 USD/CHF**.

Suppose S_t = 0.75 USD/CHF ⇒ $P_{oil\text{-}SWIT}$ (in USD) = **CHF 80** * 0.75 USD/CHF = USD 60.
That is, a barrel of oil in Switzerland is more expensive -once denominated in USD- than in the US.

Arbitrageurs/traders will buy oil in the U.S. (to export it to Switzerland) and simultaneously sell oil in Switzerland. This movement of oil will simultaneously increase the price of oil in the U.S. ($P_{oil\text{-}USA}$ ↑); decrease the price of oil in Switzerland ($P_{oil\text{-}SWIT}$ ↓); and appreciate the USD against the CHF (S_t ↓). ¶

LOOP Notes:

- LOOP gives an *equilibrium* exchange rate (EER, in the econ lit). Equilibrium will be reached when there is no trade in oil (because of pricing mistakes). That is, when the LOOP holds for oil.
- LOOP is telling us what S_t should be (in equilibrium): S_t^{LOOP}. It is not telling what S_t is in the market. It is just an *implied* rate from market prices.
- Using the LOOP, we have generated a model for S_t. (Recall that a model is an attempt to explain and predict economic phenomena.) When applied to a price index, we will call this model, *the PPP model.*
- The generated model, like all models, is a simplification of the real world. For example, we have ignored (or implicitly assumed negligible) trade frictions (transportation costs, tariffs, etc.).

Problem for the LOOP: There are many traded goods in the economy.
Solution: Work with baskets of goods that represent many goods. For example, the CPI basket (in the U.S., we use the CPI-U, which reflects spending patterns for urban consumers), which includes housing

(41%), transportation (17%), food & beverages (15%), health care (7%), recreation (6%), etc. The price of a basket is the weighted average price of the components. For example:

Price CPI-U basket = .41 x Price of housing + .17 x Price of transportation + ...

The price of the CPI basket is usually referred as the "*price level*" of an economy.

8.1.1 *Absolute Version of PPP*

The FX rate between two currencies is simply the ratio of the two countries' general price levels:

$$S_t^{PPP} = \text{Domestic Price level / Foreign Price level} = P_d / P_f \qquad \text{(Absolute PPP)}$$

Example: Law of one price for CPIs.

CPI-basket$_{USA}$ = P_{USA} = **USD 5,577**

CPI-basket$_{SWIT}$ = P_{SWIT} = **CHF 6,708**

$\Rightarrow S_t^{PPP}$ = **USD 5,577/CHF 6,708 = 0.8314 USD/CHF**.

If $S_t \neq$ **0.8314 USD/CHF**, there will be trade of the goods in the baskets.

Suppose S_t = **1.09 USD/CHF** > S_t^{PPP}.

$\Rightarrow$ P_{SWIT} (in USD) = **CHF 6,708** * **1.09 USD/CHF**

= **USD 7,311.72** > P_{USA} = **USD 5,577**

"Things" –i.e., the components in the CPI basket- are, on average, cheaper in the U.S. There is a potential profit from trading the CPI basket's components:

Potential profit: **USD 7,311.72** – **USD 5,577** = **USD 1,734.72**

Traders will do the following "pseudo-arbitrage" strategy:

1) Borrow USD
2) Buy the CPI-basket in the US (CPI-basket$_{USA}$ ↑)
3) Sell the CPI-basket, purchase in the US, in Switzerland. (CPI-basket$_{SWIT}$ ↓) => S_t^{PPP} ↑
4) Sell CHF/Buy USD (S_t (USD/CHF) ↓)
5) Repay the USD loan, keep the profits.

Note: Prices move and push S_t (market price) & S_t^{PPP} (equilibrium price) towards convergence. ¶

Under PPP, a USD buys the same amount of goods in the U.S. and in Switzerland. That is, a USD has the same *purchasing power* in the U.S. & in Switzerland. Vice versa, a CHF buys the same amount of goods in Switzerland and in the U.S.

• **Absolute PPP**: **The Real Exchange Rate**

The absolute version of the PPP theory is expressed in terms of S_t, the *nominal exchange rate*.

We can express the absolute version of the PPP relationship in terms of the *real exchange rate*, R_t. That is,

$$\mathbf{R_t} = S_t \; P_f / P_d.$$

The real exchange rate allows us to compare foreign prices, translated into domestic terms, with domestic prices. It is common to associate $\mathbf{R_t} > 1$ with a more efficient/productive domestic economy.

If absolute PPP holds $\Rightarrow \mathbf{R_t} = 1$.

Terminology: If $\mathbf{R_t}$ ↑, foreign goods become more expensive relative to domestic goods. We say there is "a *real depreciation* of the DC". Similarly, if $\mathbf{R_t}$ ↓, we say there is "a *real appreciation* of the DC."

Example: Suppose a basket –the Big Mac (sesame-seed bun, onions, pickles, cheese, lettuce, beef patty and special sauce)– costs CHF 6.70 and USD 5.36 in Switzerland and in the U.S., respectively.

P_f = **CHF 6.70**

P_d = **USD 5.36**

S_t = **1.0836 USD/CHF** $\Rightarrow P_f$ (in USD) = USD 7.26 > P_d

$\mathbf{R_t} = S_t\, P_{SWIT}/P_{US}$ = **1.0836 USD/CHF** * **CHF 6.70**/**USD 5.36** = **1.3545**

Taking the Big Mac as our basket, the U.S. is more competitive than Switzerland. Swiss prices are **35.45%** higher than U.S. prices, after taking into account the nominal exchange rate.

To bring the economy back to equilibrium –no trade on Big Macs-, we expect the USD to appreciate against the CHF. According to PPP, the USD is undervalued against the CHF:

$\Rightarrow$ Trading signal: Buy USD/Sell CHF.

Note: Obviously, we do not expect to see Swiss consumers importing Big Macs from the U.S.; but the components of the Big Mac are internationally traded. Trade would happen in the components! ¶

Indicator of under/over-valuation: $\mathbf{R_t} > 1 \Rightarrow$ FC is overvalued.

Note: In the short-run, we will not take our cars to Mexico to be repaired, because a mechanic's hour is cheaper than in the U.S. But in the long-run, resources (capital, labor) will move, likely to produce cars in Mexico to export them to the U.S. We can think of the over-/under-valuation as an indicator of movement of resources.

Remark: If S_t changes, but P_f & P_d move in such a way that $\mathbf{R_t}$ remains constant, changes in S_t do not affect firms. There is no change in real cash flows.

Aside: Economists like to work with logs. Thus, using lower case letters to denote the log of variables, x=log(X), we can rewrite the real exchange rate as:

$$\mathbf{r_t} = s_t + p_{f,t} - p_{d,t}$$

If PPP holds, then $\mathbf{r_t} = 0$. Then, $s_t = p_{f,t} - p_{d,t}$.

• **Absolute PPP**: **Real v. Nominal Exchange Rates**

Economists think that monetary variables affect nominal variables, like prices and the nominal exchange rate, S_t. But, monetary variables do not affect real variables. In this case, only relative demands and supplies affect $\mathbf{R_t}$.

For example, an increase in U.S. output relative to European output (say, because of a technological innovation) will decrease P_{US} relative to P_{EUR} ⇒ R_t ↑ (a real depreciation of the USD). On the other hand, a monetary approach to exchange rates, predicts that an increase in the U.S. money supply will increase P_{US} and, thus, an increase in S_t, but no effect on R_t.

• Absolute PPP: Does it hold?

We use a basket of goods to test PPP. To get better results, it is a good idea to use the same basket (or comparable baskets). For example, the Big Mac.

Example: The Economist's Big Mac Index, shown in Exhibit 8.1, shows the over/undervaluation of a currency relative to the USD. That is, it shows the real exchange rate, R_t - 1.

$R_t = S_t\, P_{BM,f} / P_{BM,d=US}$ (or using log notation: $r_t = s_t + p_{f,t} - p_{d,t}$)

Test: If Absolute PPP holds ⇒ $R_t = 1$ (& over/undervaluation=0!).

Exhibit 8.1: The Economist's Big Mac Index (July 2019)

The Big Mac index

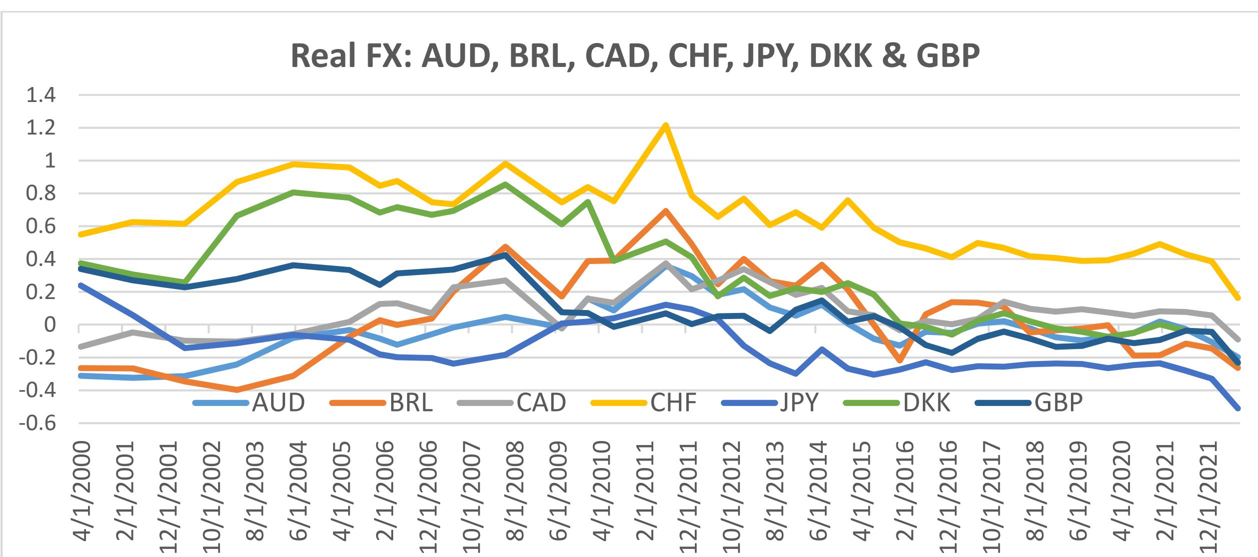
Country		2000 —— 2022	Under/over valued, %
Switzerland	Franc		35.4
Uruguay	Peso		27.8
Norway	Krone		22.9
Sweden	Krona		4.8
Denmark	Krone		0.9
United States	US$		BASE CURRENCY
Argentina	Peso		-1.0
Euro area	Euro		-1.4
Australia	A$		-4.6
Saudi Arabia	Riyal		-5.6
Israel	Shekel		-5.7
Sri Lanka	Rupee		-6.9
Costa Rica	Colón		-7.4
UAE	Dirham		-8.6
New Zealand	NZ$		-9.0
Chile	Peso		-11.4
Britain	Pound		-12.9
Kuwait	Dinar		-14.5
Canada	C$		-14.7
Czech Rep.	Koruna		-15.8

Country		2000 —— 2022	Under/over valued, %
Bahrain	Dinar		-15.9
Lebanon	Pound		-16.5
Singapore	S$		-16.6
Brazil	Real		-17.2
Nicaragua	Córdoba		-21.2
Mexico	Peso		-21.8
Colombia	Peso		-22.4
Poland	Zloty		-23.6
Honduras	Lempira		-24.5
Turkey	Lira		-25.6
South Korea	Won		-26.0
Thailand	Baht		-27.2
Qatar	Riyal		-28.3
Hungary	Forint		-29.9
Oman	Rial		-31.2
Peru	Sol		-33.2
China	Yuan		-34.0
Jordan	Dinar		-34.3
Guatemala	Quetzal		-35.8
Pakistan	Rupee		-36.8

Check: http://www.economist.com/content/big-mac-index

There are big deviations from Absolute PPP, which can vary a lot over time. See Figure 8.4 below for two R_t series (April 2000 – January 2016): CHF/USD, yellow line; and BLR/USD, orange line.

Figure 8.4: PPP – Persistent deviation from R_t=1 (2000-2022



With some exceptions, the Big-Mac tends to be more expensive in developed countries (Euro area, Australia) than in less developed countries (Egypt, South Africa, China). ¶

Empirical Fact: Price levels in richer countries are consistently higher than in poorer ones. It is estimated that a doubling of income per capita is associated with a 48% increase in the price level. This empirical fact is called the *Penn effect*. Many explanations, the most popular: The *Balassa-Samuelson (BS) effect*.

• Absolute PPP: Qualifications

The big deviations from absolute PPP are usually attributed to different reasons:

(1) PPP emphasizes only trade and price levels. Other financial, economic, political factors are ignored.

(2) Absence of trade frictions. This is an implicit assumption: No tariffs, no quotas, no transactions costs. Realistic? It is estimated that transportation costs add 7% to the price of U.S. imports of meat and 16% to the import price of vegetables. Some products are heavily protected, even in the U.S. For example, peanut imports are subject to a tariff between 131.8% (for shelled peanuts) and 163.8% (for unshelled peanuts).

(3) Perfect competition. Imperfect competition, usually related to (2) can create price discrimination. For example, U.S. pharmaceuticals sell the same drug in the U.S. and in Canada at different prices.

(4) Instantaneous adjustments. Another implicit PPP assumption, related to another trade friction. Not realistic. Trade takes time and it also takes time to adjust contracts. Think of PPP as *long-run* model.

(5) PPP assumes P_f and P_d represent the same basket, not the usual situation for CPI baskets. This is why the Big Mac is a popular basket: it is standardized around the world with an easy to get price.

(6) Internationally non-traded (NT) goods (~50%-60% of GDP) –i.e., haircuts, hotels, restaurants, home & car repairs, medical services, real estate, etc. NT goods have a big weight on the CPI basket.

Suppose the proportion of NT in the price basket is α_N. Then, using log price notation, we can write the (log) price index as:

$$p_t = \alpha_N \, p_{NT,t} + (1 - \alpha_N) \, p_{T,t}$$

where $p_{NT,t}$ is the log price of NT goods and $p_{T,t}$ is the log price of taded (T) goods. If PPP holds for traded goods, $s_t + (p_{f,T,t} - p_{d,T,t}) = 0$, then, after some algebra:

$$\mathbf{r_t} = \alpha_N \, (p_{f,NT,,t} - p_{d,NT,t} + s_t).$$

Does PPP hold for NT goods?

(7) The NT sector also has an effect on the price of traded goods. For example, rent, distribution and utilities costs affect the price of a Big Mac. (It is estimated that 25% of Big Mac's cost is due to NT goods.)

• Borders Matter

You may look at the Big Mac Index and think: "No big deal: there is also a big dispersion in prices within the U.S., within Texas, and, even, within Houston!" It is true that prices vary within the U.S. (or within any country). For example, in 2015, the price of a Big Mac (and Big Mac Meal) in New York was USD 5.23 (USD 7.45), in Texas was USD 4.39 (USD 6.26) and in Mississippi was USD 3.91 (USD 5.69).

Engel and Rogers (1996) computed the variance of LOOP deviations for city pairs within the U.S., within Canada, and across the border. They found that distance between cities within a country matter, but the border effect is very significant. To explain the difference between prices across the border using the estimate distance effects within a country, they estimate the U.S.-Canada border should have a width of 75,000 miles!

This huge estimate of the implied border width between the U.S. and Canada has been revised downward in subsequent studies, but a large positive border effect remains.

• Balassa-Samuelson Effect
Balassa (1964) and Samuelson (1964) developed a general equilibrium model of the real exchange rate (BS model). The model explains the above mentioned empirical fact: richer countries have consistently higher prices.

Labor costs affect all prices. We expect average prices to be cheaper in poor countries than in rich ones because labor costs are lower. Rich countries have higher productivity, and higher wages, in the traded-goods sector than poor countries do. In the NT goods sector, productivity is similar.

But, because of competition for labor, wages in NT goods and services are also higher in rich countries. Then, overall prices are lower in poor countries. For example, the productivity of McDonald's employees around the world is likely very similar, but the wages are not. In 2000, a typical McDonald's worker in the U.S. made USD 6.50/hour, while in China made USD 0.42/hour. This difference in NT costs may partly explain over/under-valuations when we compare currencies from developed to less developed countries.

Again, standard applications of PPP, like in the Big Mac example above, will not be very informative. We need to "adjust" prices to incorporate the effect of GDP per capita in the price level.

Practitioners tend to incorporate the Balassa-Samuelson effect into PPP calculations in a straightforward manner. Suppose we want to adjust Big Mac PPP-implied exchange rates. Then:

1) Estimate a regression using Big Mac Prices (in USD, $P_{BM,t}$) as the dependent variable against GDP per capita (GDP_p). That is, run the following regression:

$$P_{BM,t} = \alpha + \beta \text{ GDP\_p}_t + \varepsilon_t$$

2) Compute fitted values (GDP-adjusted Big Mac Prices). That is,

$$\hat{P}_{BM,GDP\text{-}adjusted} = \hat{\alpha} + \hat{\beta} \text{ GDP\_per\_capita}_t$$

Based on the GDP-adjusted Big Mac Prices, re-estimate the PPP implied over/under-valuation:

GDP-adjusted over/under valuation: (BM Price/$\hat{P}_{BM,GDP\text{-}adjusted}$) – 1

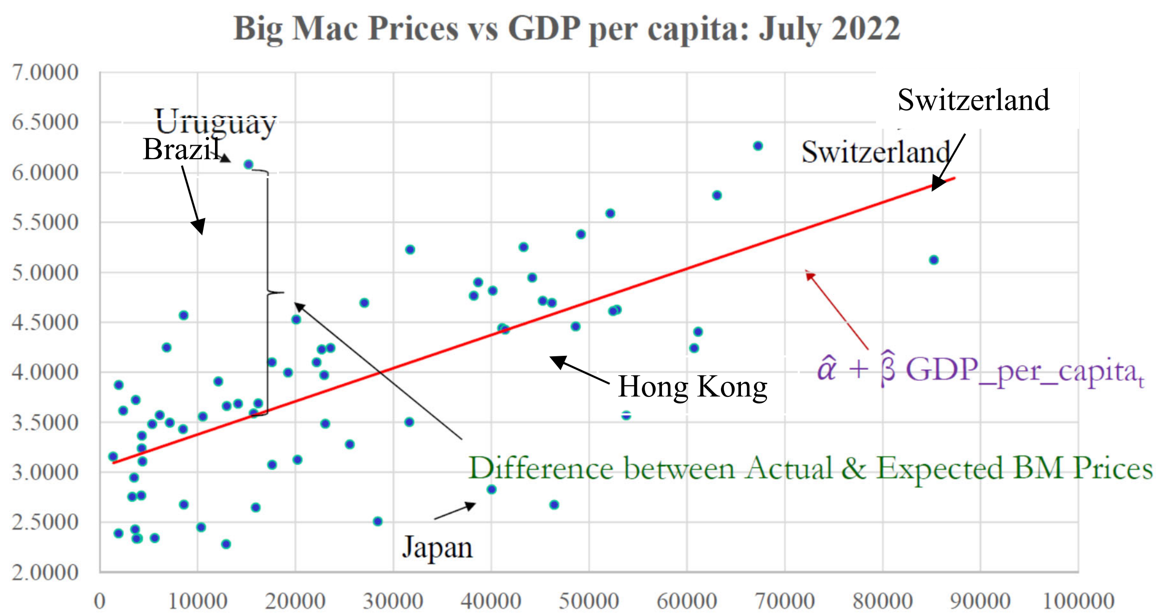
The regression line tells us what the "expected price" in a country is, once we take into consideration its GDP level.

Using data from The Economist for July 2022, we estimated the above regression:

$$\hat{P}_{BM,GDP\text{-}adjusted} = 3.045895 + 0.0000332 * \text{GDP\_per\_capita}_t$$

Figure 8.5 shows the regression line using data from in July 2022:

Figure 8.5: PPP – Regression to Adjust Big Mac Prices by GDP per capita



Now, using the computed red line above, we calculate the "Expected BM prices, given the GDP of a given country." For example, we compute the above value for Uruguay. Uruguay's GDP per capita in July 2022 was **USD 15,169.153**. Then,

$\hat{P}_{BM,GDP\text{-}adj}$ (Uruguay) = 3.045895 + 0.0000332 * **15,169.153** = **3.549511**

That is, the expected USD Big Mac price, in Uruguay, given its GDP per capita, was **USD 3.55.** Since the observed local BM price was UYU 255, which translates to **USD 6.08** (= UYU 255/**41.91 UYU/USD**), then the *GDP-adjusted over/under valuation* was:

6.08 / **3.549511** – 1 = **71.29%** (**71.29%** overvalued)

On the other hand, we have Japan; which according to the adjusted index, its currency is undervalued by **35%**. That is, these adjustments to PPP implied exchange rates can be significant. See Exhibit 8.2 below from *The Economist* for July 2011.

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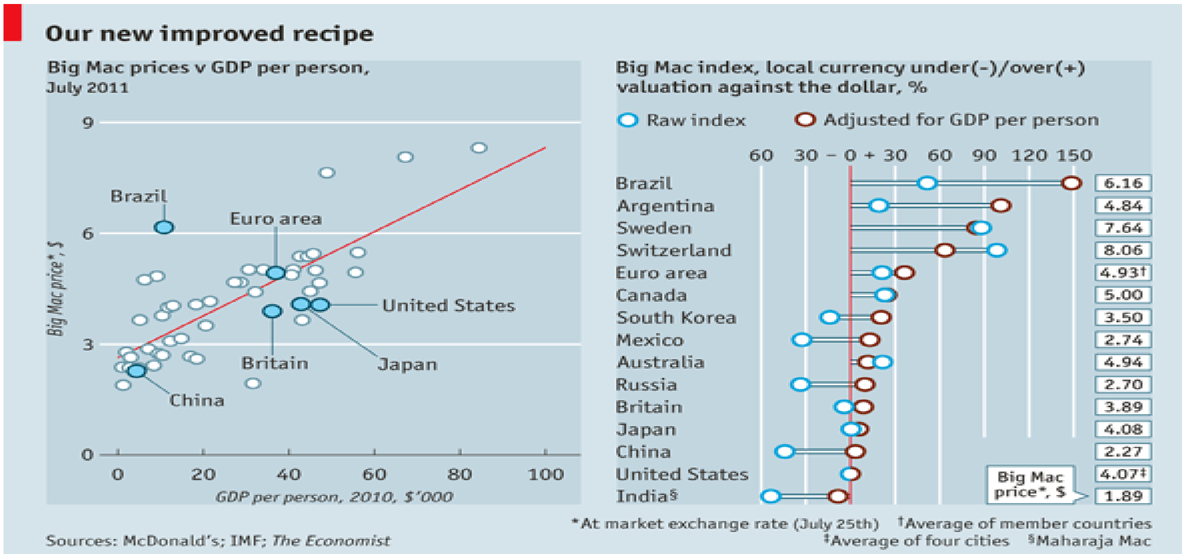
Exhibit 8.2: The Economist's Adjusted-Big Mac Index (January 2011)



The Balassa-Samuelson effect can explain (or partially explain) why absolute PPP does not hold between a developed country and a less developed country, for example, after correcting for the BS effect, China's currency is no longer undervalued. But the BS effect cannot explain why PPP does not hold among developed countries (say Switzerland and the U.S.) or among less developed countries (say, Brazil and Argentina).

• **Pricing-to-market**

Krugman (1987) offers an alternative explanation for the strong positive relationship between GDP and price levels: *Pricing-to-market* –i.e., price discrimination. Based on price elasticities, producers discriminate: the same exact good is sold to rich countries (lower price elasticity) at higher prices than to poorer countries (higher price elasticity). For example, Alessandria and Kaboski (2008) report that U.S. exporters, on average, charge the richest country a 48% higher price than the poorest country.

That is, the price of T goods consumed in the domestic (home) market, P_{TH}, is different from the T goods consumed in the foreign market, P_{TF}. Using log price notation: $p_{TH,t} \neq p_{TF,t}$.

Again, pricing-to-market struggles to explain why PPP does not hold among developed countries with similar incomes. For example, Baxter and Landry (2012) report that IKEA prices deviate 16% from the LOOP in Canada, but only 1% in the U.S. Similarly, The Economist (2019) reports that Pret a Manger, a coffee-shop chain, sells the same items in Boston at higher prices than in London, for instance, an egg-sandwich costs GBP 1.79 (USD 2.15) in the U.K. and USD 4.99 in the U.S.

• **Absolute PPP: Empirical Evidence**:

Several tests of the absolute version have been performed. The absolute version of PPP, in general, fails (especially, in the short run), even when using the same basket or the same good. No surprise here, see the Big Mac example above, where $\mathbf{R_t} \neq 1$. Trade frictions, especially transportation and distribution costs, are considered a major problem for absolute PPP.

8.1.2 Relative PPP

A more flexible version of PPP: The rate of change in the prices of products should be similar when measured in a common currency, as long as trade frictions are unchanged. Thus, Relative PPP addresses the assumption of no trade frictions. (All the other qualifications still apply!)

The following formula states the relative version of PPP:

$$e_{f,T}^{PPP} = \frac{S_{t+T}^{PPP} - S_t}{S_t} = \frac{(1 + I_{d,t+T})}{(1 + I_{f,t+T})} - 1 \qquad \text{(Relative PPP)},$$

where
$e_{f,T}^{PPP}$ = percentage change in the value of FC from t to t+T.
$I_{f,t+T}$ = foreign inflation rate from t to t+T
$I_{d,t+T}$ = domestic inflation rate from t to t+T.

Linear approximation (from a 1st-order Taylor series): $e_{f,T}^{PPP} \approx I_{d,t+T} - I_{f,t+T}$

Example: Suppose that, from t=0 to t=1, prices increase 10% in Mexico relative to those in Switzerland. Then, $S_{MXN/CHF,t}$ should increase 10%; say, from S_0=9 MXN/CHF to S_1=9.9 MXN/CHF. If, at t=1, S_1=11 MXN/CHF > $S_{t=1}^{PPP}$ = 9.9 MXN/CHF, then according to Relative PPP the CHF is overvalued. ¶

Example: Forecasting S_t (USD/ZAR) using PPP (ZAR=South Africa).
It's 2015. You have the following information:
$CPI_{US,2015}$ = 104.5,
$CPI_{SA,2015}$ = 100.0, S_{2015} =.2035 USD/ZAR.

You are given the 2016 CPI's forecast for the U.S. and SA: E[$CPI_{US,2016}$]=110.8, E[$CPI_{SA,2016}$]=102.5.

You want to forecast S_{2016} using the relative (linearized) version of PPP.
E[$I_{US-2016}$] = (110.8/104.5) – 1 = .06029
E[$I_{SA-2016}$] = (102.5/100) – 1 = .025

E[S_{2016}] = S_{2015} * (1 + E[I_{US}] – E[I_{SA}]) = .2035 USD/ZAR * (1 + .06029 – .025) = .2107 USD/ZAR. ¶

• Relative PPP: Implications
(1) Under relative PPP, **R_t** remains constant (it can be different from 1!).
(2) Relative PPP does not imply that S_t is easy to forecast.
(3) Without relative price changes, a multinational corporation faces no real operating exchange risk (as long as the firm avoids fixed contracts denominated in foreign currency).

• Relative PPP: Absolute versus Relative
Absolute PPP compares price levels, while Relative PPP compares price changes (or movements). Under Absolute PPP prices are equalized across countries, but under Relative PPP exchange rates move by the same amount as the inflation rate differential (original prices can be different).

Relative PPP is a weaker condition than the absolute one: **R_t** can be different from 1.

Example: Absolute vs Relative

Absolute PPP: "*A mattress costs GBP 200 (= USD 320) in the U.K. and BRL 800 (=USD 320) in Brazil –i.e., same cost in both countries*." (S_t =1.6 USD/GBP & S_t =0.4 USD/BRL)

Relative PPP: "*U.K. inflation was 2% while Brazilian inflation was 8%. Meanwhile, the BRL depreciated 6% against the GBP. Then, relative cost comparison remains the same*." ¶

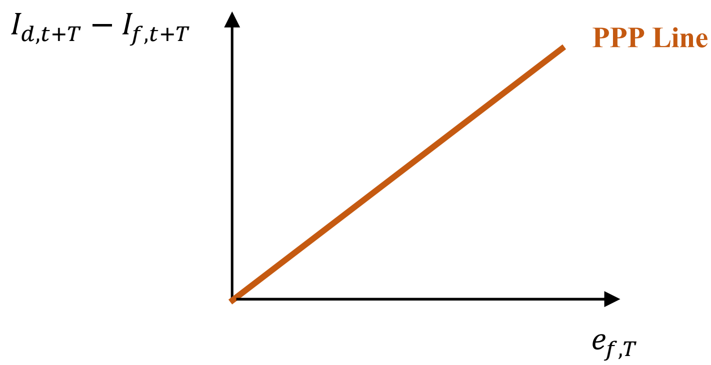
• Relative PPP: General Evidence

Key: On average, what we expect to happen, $e_{f,t}^{PPP}$, should happen, $e_{f,T}$

⇒ Q: Is, on average, $e_{f,T}^{PPP} \approx I_{d,t+T} - I_{f,t+T} = e_{f,T}$?

Under PPP, we should see $e_{f,T}$ and $I_{d,t+T} - I_{f,t+T}$ aligned around a 45º line, like in Graph 8.1.

Figure 8.6: PPP Line



1. Visual Evidence

Figure 8.7 plots (I_d-I_f) between Japan and the U.S. against e_f JPY/USD), using 1974-2022 monthly data.

Figure 8.7: PPP Line for the JPY/USD (1974-2022)?

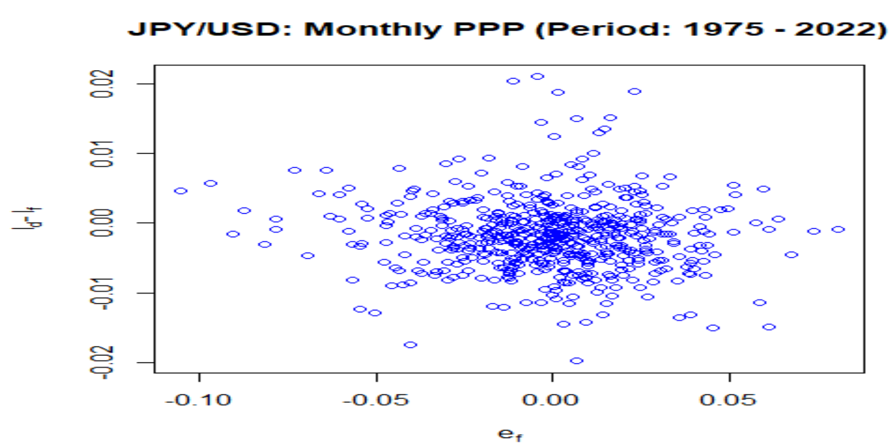
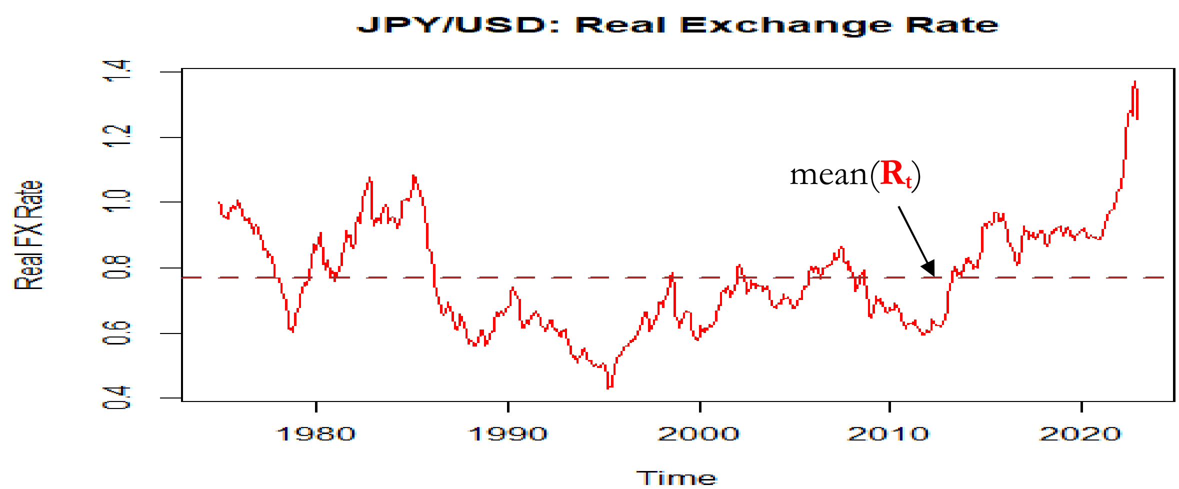


Figure 8.8-A plots R_t between Japan and the U.S. It can be used to check if R_t is constant (ideally, under absolute PPP, close to 1, but we do not have prices, but indices. R_t is arbitrary set to 1 in Jan 1971):

Figure 8.8-A: Real Exchange Rate between Japan and USA (1974-2022)

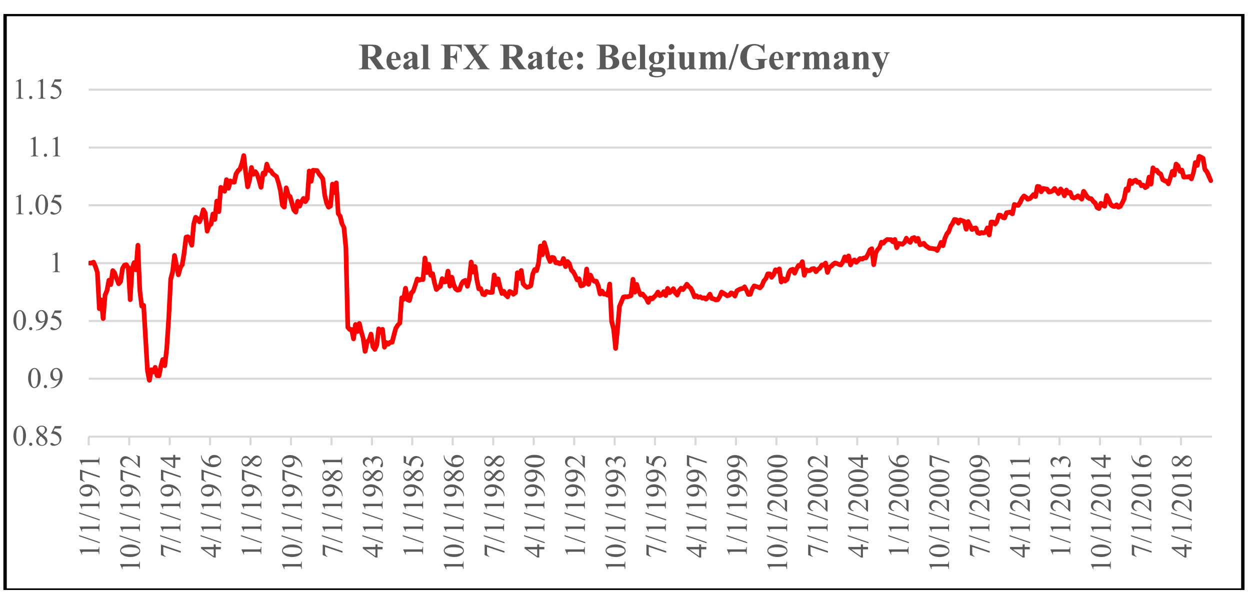


Clearly, $\mathbf{R_t}$ is not constant! In general, we have some evidence for *mean reversion* for $\mathbf{R_t}$ in the long run. Loosely speaking, $\mathbf{R_t}$ moves around some mean number, which we associate with a *long-run PPP parity* (for the JPY/USD the average $\mathbf{R_t}$ is 0.77). But, the deviations from the long-run PPP parity are very *persistent* –i.e., very slow to adjust. Note that the deviations from long-run PPP parity can be big (up to 66% from the mean) and happen in every decade.

Economists usually report the number of years that a PPP deviation is expected to decay by 50% (the *half-life)* is in the range of 3 to 5 years for developed currencies. Very slow!

Note that even for currencies with a fixed exchange rate, like Belgium and Germany, $\mathbf{R_t}$ is not constant, as illustrated in Figures 8.8-B.

Figure 8.8-B: Real Exchange Rate between Belgium and Germany (1970-2019)



2. Statistical Evidence

Let's look at the usual descriptive statistics for $(I_d - I_f)_t$ and $e_{f,t}$ using the 1974-2022 monthly data used above. For the JPY/USD, they have similar means, but quite different standard deviations (look at the very different minimum and maximum stats). A simple t-test for equality of means (t-test=-0.34) cannot reject the null hypothesis of equal means, which is expected given the large SDs, especially for $e_{f,t}$.

	I_{JP}	I_{US}	I_{US}-I_{JP}	$e_{f,T}$ (JPY/USD)	R_t
Mean	**0.00125**	**0.00303**	**-0.00179**	**-0.00139**	**0.7676**
SD	**0.00485**	**0.00322**	**0.00502**	**0.02622**	**0.1582**
Min	-0.01095	-0.01786	-0.01981	-0.08065	0.4295
Median	0.00102	0.00266	-0.00184	0.00022	0.7477
Max	0.02558	0.01420	0.02104	0.08066	1.3717

If we think of the average over the whole sample, as a long-run value, we cannot reject PPP in the long-run! But, the average relation over the whole sample is not that informative, especially with such a big SD. We are more interested in the short-run, in the contemporaneous relation between $e_{f,T}$ and $(I_d - I_f)_t$. That is, what happens to $e_{f;T}$ when $(I_d - I_f)_t$ changes?

To test the contemporaneous relation we have a more formal test, a regression:

$$e_{f,T} = (S_{t+T} - S_t)/S_t = \alpha + \beta\,(I_d - I_f)_T + \varepsilon_T, \quad \text{(where } \varepsilon_T \text{ is the regression error, } E[\varepsilon_T]=0).$$

The null hypothesis is: H_0 (Relative PPP holds): **$\alpha=0$** & **$\beta=1$**

H_1 (Relative PPP does not hold): **$\alpha\neq 0$** and/or **$\beta\neq 1$**

Tests: t-test (individual tests on the estimated α and β) and F-test (joint test):

(1) t-test = $t_{\theta=\theta_0} = [\hat{\theta} - \theta_0] / \text{S.E.}(\hat{\theta}) \sim t_v$ ($v = N - K$ = *degrees of freedom*).

(2) $F = \dfrac{[\text{RSS}(H_0) - \text{RSS}(H_1)]/J}{\text{RSS}(H_1)/(N-K)} \sim F_{J,N-K}$ (J = # of restrictions in H_0).

Notation for tests:

- $\theta = (\alpha, \beta)$
- $\hat{\theta}$ = Estimated θ
- e_t = residuals = $e_{f,T} - [\,\hat{\alpha} + \hat{\beta}\,(I_d - I_f)_t\,]$
- H_0 (theory is true): $\theta = \theta_0$
- N = # of observations
- K = # of parameters in our model, in the PPP case 2: (α, β)
- RSS = Residual Sum of Squares = $\Sigma_t (e_t)^2$.
- J = # of restrictions in H_0, in the PPP case 2: **$\alpha = 0$** & **$\beta = 1$** .
- α = significance level –most popular, α =.05 (5 %).
- t_v = t-distribution with *v degrees of freedom* (df). (When $v > 30$, it follows a normal).
- $F_{J,N-K}$ = F-distribution with J df in the numerator and N-K df in the denominator.

Rules for tests:

If $|\text{t-test}| > |t_{v,\alpha/2}|$, reject H_0 at the α level. (When α = .05 & $v > 30$, $\mathbf{t_{.025}}$ = **1.96**.)

If F-test > $F_{J,N-K,\alpha}$, reject H_0 at the α level. (When α = .05 & (N – K) > 300, $\mathbf{F_{2,300+,.05}} \approx$ **3**.)

Example: We want to test relative PPP for the JPY/USD exchange rate (we use α = .05). We use the monthly Japanese and U.S. data from the graph (1/1975 - 12/2022). We fit the following regression:

$$e_{f,T} \text{ (JPY/USD)} = (S_t - S_{t-1})/S_{t-1} = \alpha + \beta\ (I_{JAP} - I_{US})_t + \varepsilon_t .$$

$R^2 = 0.005621$
Standard Error (σ) = .02617
F-stat (slopes=0 –i.e., **β=0**) = 3.244 (*p-value* = 0.07219)
F-test (H_0: **α=0** and **β=1**) = **19.185** (*p-value*: lower than 0.0001) ⇒ reject at 5% level (**$F_{2,574,.05}$** = **3.01**)
Observations = 576

	Coefficients	Stand Error	t Stat	P-value
Intercept ($\hat{\alpha}$)	-0.00209	0.001157	-1.804	0.0717
($I_{JAP} - I_{US}$) ($\hat{\beta}$)	-0.39148	0.217343	-1.801	0.0722

Note: You can find this example in my homepage: www.bauer.uh.edu/rsusmel/4386/ppp-example.xls

Let's test H_0, using t-tests ($t_{580,.025}$ = **1.96** –when N – K > 30, $t_{.05}$ = **1.96**):
$t_{\alpha=0}$ (t-test for **α = 0**): (-0.00209 – 0)/ 0.00116 = -1.804 (*p-value* = .072) ⇒ cannot reject at the 5% level
$t_{\beta=1}$ (t-test for **β = 1**): (-0.39148 – 1)/ 0.21734 = -6.402 (*p-value* < .00001) ⇒ reject at the 5% level

Let's test H_0, using the F-test ($F_{2,574,.05}$ = **3.01**):
RSS(H_0) = 0.4211
RSS(H_1) = 0.3930
J = 2; (N – K) = 574
F-test = {[0.4211 – 0.3930]/2}/{0.3930/579} = **19.185** > **3.015** ⇒ reject at the 5% level.

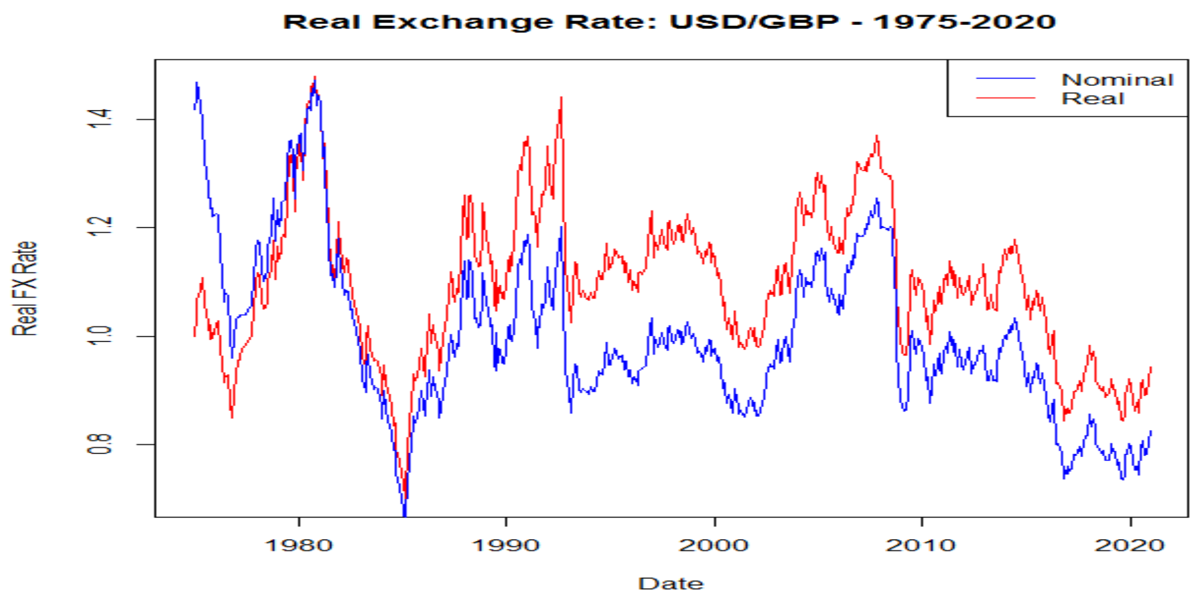
Regression Notes:
- If we look at the R^2, the variability of monthly ($I_{JAP} - I_{US}$) explain very little, 0.01%, of the variability of monthly $e_{f,T}$.
- We can modify the regression to incorporate the Balassa-Samuelson effect, by incorporating GDP differentials. For example:

$$e_{f,T} \text{ (JPY/USD)} = \alpha + \beta\ (I_{JAP} - I_{US})_t + \delta\ (GDP_{cap,JAP} - GDP_{cap,US})_t + \varepsilon_t.$$ ¶

Relative PPP tends to be rejected in the short-run (like in the example above). In the long-run, there is a debate about its validity. As mentioned above there is some evidence of (slow) mean reversion. In the long-run, inflation differential matter: Currencies with high inflation rate differentials tend to depreciate.

• PPP: R_t and S_t
Research shows that **R_t** is much more variable when S_t is allowed to float. **R_t**'s variability tends to be highly correlated with S_t's variability. This finding comes from Mussa (1986).

Figure 8.9: Nominal and Real Exchange Rate for the USD/GBP (1975-2020)



In Figure 8.9, above, we see the finding of Mussa (1986) for the USD/GBP exchange rate: After 1973, when floating exchange rates were adopted, $\mathbf{R_t}$ moves like S_t. As a check to the visual evidence: the monthly volatility of changes in $\mathbf{R_t}$ is 2.706% and the monthly volatility of changes in S_t is 2.622%, with a correlation coefficient of .983. Almost the same!

Recall that economists tend to think that nominal variables can affect nominal variables, but not real variables. The above graph shows that S_t moves like $\mathbf{R_t}$, which we think is affected by real factors. We can incorporate real factors into the determination of S_t (using the definition of $\mathbf{R_t}$, we solve S_t):

$$S_t = \mathbf{R_t}\ P_d / P_f.$$

Now, we have S_t affected by real factors (through $\mathbf{R_t}$) and nominal factors (through P_d / P_f).

• PPP: Sticky Prices

From the above USD/GBP graph, which is representative of the usual behavior of $\mathbf{R_t}$ and S_t, we infer that price levels play a minor role in explaining the movements of $\mathbf{R_t}$ (& S_t). Prices are *sticky/rigid* –i.e., they take a while to adjust to shocks/disequilibria.

A potential justification for the implied price rigidity: NT goods. Price levels include traded and NT goods; traded-goods should obey the LOOP. But, Engel (1999) and others report that prices are sticky also for traded-goods (measured by disaggregated producer price indexes). A strange result for many of us that observe gas prices change frequently!

Possible explanations:

(a) Contracts

Prices cannot be continuously adjusted due to contracts. In a stable economy, with low inflation, contracts may be longer. We find that economies with high inflation (contracts with very short duration) PPP deviations are not very persistent.

(b) Mark-up adjustments
There is a tendency of manufacturers and retailers to moderate any increase in their prices in order to preserve their market share. For example, changes in S_t are only partially transmitted or *pass-through* to import/export prices. The average ERPT (exchange rate pass-through) is around 50% over one quarter and 64% over the long run for OECD countries (for the U.S., 25% in the short-run and 40% over the long run). The average ERPT seems to be declining since the 1990s. Income matters: ERPT tends to be bigger in low income countries (2-4 times bigger) than in high countries.

(c) Repricing costs (*menu costs*)
It is expensive to adjust continuously prices; in a restaurant, the repricing cost is re-doing the menu. For example, Goldberg and Hallerstein (2007) estimate that the cost of repricing in the imported beer market is 0.4% of firm revenue for manufacturers and 0.1% of firm revenue for retailers.

(d) Aggregation
Q: Is price rigidity a result of aggregation –i.e., the use of price index? Empirical work using detailed micro level data –say, same good (exact UPC barcode!) in Canadian and U.S. grocery stores– show that on average product-level $\mathbf{R_t}$ –i.e., constructed using the same traded goods– move closely with S_t . But, individual micro level prices show a lot of idiosyncratic movements, mainly unrelated to S_t : Only 10% of the deviations from PPP are accounted by S_t .

• PPP: Puzzle
The fact that no single model of exchange rate determination can accommodate both the high persistent of PPP deviations and the high correlation between R_t and S_t has been called the "*PPP puzzle.*" See Rogoff (1996).

• PPP: Summary of Empirical Evidence
- R_t and S_t are highly correlated, domestic prices (even for traded-goods) tend to be sticky.
- In the short run, PPP is a very poor model to explain short-term exchange rate movements.
- PPP deviation are very persistent. It takes a long time (years!) to disappear.
- In the long run, there is some evidence of mean reversion, though very slow, for R_t. That is, $\mathbf{S_t^{PPP}}$ has long-run information: Currencies that consistently have high inflation rate differentials –i.e., (I_d-I_f) positive- tend to depreciate.

The long-run interpretation for PPP is the one that economist like and use. PPP is seen as a benchmark, a figure towards which the current exchange rate should move.

• Calculating $\mathbf{S_t^{PPP}}$ (Long-Run FX Rate)
We want to calculate $S_t^{PPP} = \frac{P_{d,t}}{P_{f,t}}$ over time. Steps:

(i) Divide and multiply $\mathbf{S_t^{PPP}}$ by $S_0^{PPP} = \frac{P_{d,0}}{P_{f,0}}$ (where $t = 0$ is our starting point or base year).

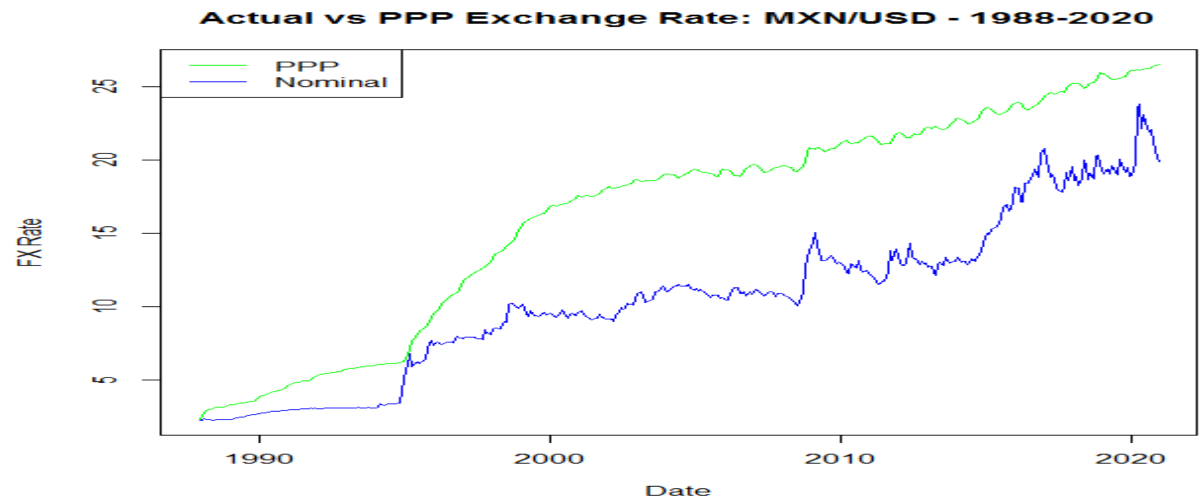
(ii) After some algebra,

$$\mathbf{S_t^{PPP}} = S_{t=0}^{PPP} * [\frac{P_{d,t}}{P_{d,0}}] * [\frac{P_{f,0}}{P_{f,t}}]$$

By assuming $S_{t=0}^{PPP} = S_0$, we can plot $\mathbf{S_t^{PPP}}$ over time. (Note: $S_{t=0}^{PPP} = S_0$ assumes that at $t = 0$, the economy was in *equilibrium*. This may not be true. That is, be careful when selecting a base year.)

Figure 8.10 plots S_t^{PPP} and S_t for the MXN/USD exchange rate during the 1987-2020 period. During the sample, Mexican inflation rates were consistently higher than U.S. inflation rates –actually, 322% higher during the sample period). Relative PPP predicts a consistent appreciation of the USD against the MXM.

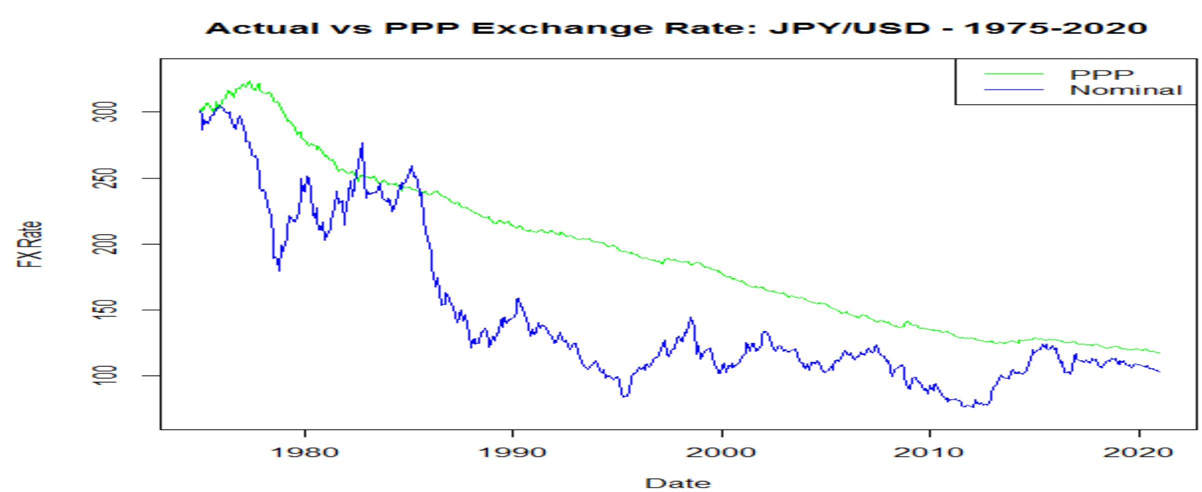
Figure 8.10: S_t^{PPP} and S_t for the MXN/USD (1987-2020)



In the short-run, Relative PPP is missing the target, S_t. But, in the long-run, PPP gets the trend right. (As predicted by PPP, the high Mexican inflation rates differentials against the U.S depreciate the MXN against the USD.)

Similar behavior is observed for the JPY/USD, as shown in Figure 8.11. The inflation rates in the U.S. have been consistently higher than in Japan (57% higher during the period), then, according to Relative PPP, the USD should depreciate against the JPY. PPP gets the long term trend right, but misses S_t in the short-run.

Figure 8.11: S_t^{PPP} and S_t for the JPY/USD (1971-2019)



Note that in both graphs, S_t^{PPP} is smoother than S_t.

• PPP: Summary of Applications

◦ Equilibrium ("*long-run*") exchange rates. A CB can use $\mathbf{S_t^{PPP}}$ to determine intervention bands.
◦ Explanation of S_t movements ("currencies with high inflation rate differentials tend to depreciate").
◦ Indicator of competitiveness or under/over-valuation: $\mathbf{R_t} > 1 \Rightarrow$ FC is overvalued (& Foreign prices are not competitive).
◦ International GDP comparisons: Instead of using S_t, $\mathbf{S_t^{PPP}}$ is used. (An additional advantage: since $\mathbf{S_t^{PPP}}$ is smoother, GDP comparisons will not be subjected to big swings.) For example, per capita GDP (World Bank figures, in 2017) are reported below in Table 8.1:

Table 8.1: GDP per capita in Nominal and PPP Prices (in USD) - 2017

Country	**GDP per capita (in USD) - 2017**	
	Nominal	*PPP*
Luxembourg	104,103	103,745
USA	59,532	59,532
Japan	38,428	43,279
Costa Rica	11,631	17,044
Brazil	9,821	15,483
Lebanon	8,524	14,676
China	**8,827**	**16,807**
India	1,937	7,056
Ethiopia	767	1,899

Example: Nominal vs PPP - Calculations for China
Data:
Nominal GDP per capita: CNY 59,670.52
S_t= 0.14792 USD/CNY;
$\mathbf{S_t^{PPP}}$ = **0.2817 USD/CNY** ⇒ "goods in the U.S. are 51.58% more expensive than in China."

- Nominal GDP_cap (USD)= CNY 59,670.52 * 0.1479 USD/CNY = **USD 8,827**
- PPP GDP_cap (USD)= CNY 59,670.52 * **0.2817 USD/CNY** = **USD 16,807**. ¶

Chapter 8 - Appendix – Taylor Series

Definition: Taylor Series
Suppose *f* is an infinitely often differentiable function on a set ***D*** and $c \in \boldsymbol{D}$. Then, the series

$$T_f(x, c) = \Sigma_n \,[\, f^{(n)}(c)/n!\,]\,(x - c)^n$$

is called the (formal) *Taylor series* of *f* centered at, or around, *c*.

Note: If c=0, the series is also called *MacLaurin Series.*

Taylor Series Theorem

Suppose $f \in C^{n+1}([a, b])$ -i.e., *f* is *(n+1)*-times continuously differentiable on *[a, b]*. Then, for $c \in [a,b]$ we have:

$$f(x) = T(x,c) + R = \frac{f(c)}{0!}(x-c)^0 + \frac{f^{/}(c)}{1!}(x-c)^1 + \frac{f^{//}(c)}{2!}(x-c)^2 + \ldots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + R$$

where $R_{n+1}(x) = \frac{1}{n!}\int f^{(n+1)}(p)(x-p)^n\,dp$

In particular, the $T_f(x, c)$ for an infinitely often differentiable function f converges to f iff the remainder $R_{(n+1)}(x)$ →converges to 0 as $n \to \infty$.

Example: 1st-order Taylor series expansion, around c=1, of $f(x)=5+2x + x^2$

$f(x) = 5+2x + x^2$ $\quad$ $f'(x_0=1) = 8$
$f'(x) = 2 + 2x$ $\quad$ $f'(x_0=1) = 4$
$f''(x) = 2$ $\quad$ $f''(x_0=1) = 2$
$f'''(x) = 0$ $\quad$ $f'''(x_0=1) = 0$

=> 1st-order Taylor's series formula (n=1):

$f(x) \approx T(x; c) = 8 + 4 (x-1) = 4 + 4x$

• Now, for the Relative PPP approximation, we use a Taylor series expansion, $T_f(x, c)$, for a bivariate series:

$$T(x, y, c, d) = \frac{f(c,d)}{0!}(x-c)^0 (y-d)^0 + \frac{f_x(c,d)}{1!}(x-c)^1 + \frac{f_y(c,d)}{1!}(y-d)^1 +$$

$$+ \frac{1}{2!}[f_{xx}(c,d)(x-c)^2 + f_{xy}(c,d)(x-c)(y-d) + f_{yy}(c,d)(y-d)^2] + \ldots$$

Example: Taylor series expansion, around d=c=0, of $f(x,y) = [(1+x)/(1+y)] -1$

$f(x,y) = [(1+x)/(1+y)] -1$ $\quad$ $\Rightarrow f(c=0,d=0) = [(1+0)/(1+0)] - 1 = 0$
$f_x = 1/(1+y)$ $\quad$ $\Rightarrow f_x(c=0,d=0) = 1$
$f_x = (-1)(1+x)/(1+y)^2$ $\quad$ $\Rightarrow f_y(c=0,d=0) = -1$

=> 1st-order Taylor's series formula:

$f(x,y) \approx T(x,y; c,d) = 0 + 1 (x-0) + (-1) (y-0) = x - y$

Application to Relative PPP: $e_{f,T}^{PPP} = [(1 + I_d)/(1 + I_f)] - 1 \quad \approx (I_d - I_f)$ ¶

Chapter 8 – Measuring the Role of NT Goods

We go back to the log definition of the real exchange rate, r_t:

$$\mathbf{r_t} = s_t + p_{f,t} - p_{d,t}$$

The log price index, p_t, is influenced by NT goods and T goods. Suppose the proportion of NT in the price basket is α_N. Then, we can write the (log) price index as:

$$p_t = \alpha_N\, p_{NT,t} + (1 - \alpha_N)\, p_{T,t}\,,$$

where $p_{NT,t}$ is the log price of NT goods and $p_{T,t}$ is the log price of Traded goods.

After some algebra, and assuming α_N, is the same at home and in the foreign country, we get:

$$\mathbf{r_t} = s_t + p_{f,t} - p_{d,t} = s_t + \alpha_N\, p_{f,,NT,t} + (1 - \alpha_N)\, p_{f,,T,t} - \alpha_N\, p_{d,NT,t} - (1 - \alpha_N)\, p_{d,T,t}$$
$$= s_t + \alpha_N\, (p_{f,NT,t} - p_{d,NT,t}) + (1 - \alpha_N)\, (p_{f,,T,t} - p_{d,T,t})$$
$$= s_t + (p_{f,T,t} - p_{d,T,t}) + \alpha_N\, \{(p_{f,NT,,t} - p_{f,T,t}) - (p_{d,NT,t}) - p_{d,T,t})\}$$

That is, the real exchange rate is affected by traded and NT goods. If PPP holds for traded goods, $s_t + (p_{f,T,t} - p_{d,T,t}) = 0$, then, the real exchange is a function of the relative prices of NT goods to traded goods in foreign and domestic markets:

$$\mathbf{r_t} = \alpha_N \{(p_{f,NT,,t} - p_{f,T,t}) - (p_{d,NT,t} - p_{d,T,t})\}$$

It is possible to introduce into the model, domestic (home) production and consumption of traded goods. In this case, we define the traded goods log price as a weighted sum of the log prices of traded goods produced (and consumed) at home, $p_{TH,t}$, and traded goods produced in the foreign country and consumed in the home market, $p_{TF,t}$:

$$p_{T,t} = (\gamma/2)\, p_{TH,t} + (2 - \gamma)/2\; p_{TF,t}$$

where $(\gamma/2)$ is the proportion of traded good consumed in the home country that is produced in the home country. There is a "*home bias*" in consumption if $\gamma > 1$. Similar definition applies to foreign T goods prices.

$$\mathbf{r_t} = s_t + \alpha_N (p_{f,NT,t} - p_{d,NT,t}) + (1 - \alpha_N)\,(p_{f,,T,t} - p_{d,T,t})$$
$$= s_t + \alpha_N (p_{f,NT,t} - p_{d,NT,t}) + (1 - \alpha_N)\; \{[(\gamma_f/2)\, p_{f,TH,t} + (2 - \gamma_f)/2\; p_{f,TF,t}] - [(\gamma_d/2)\, p_{d,TH,t} + (2 - \gamma_d)/2\; p_{d,TF,t}]\}$$

A home bias can affect PPP. If we assume no home bias (γ =1), then the real exchange rate is also a function of pricing to market in traded goods, in both markets:

$$\mathbf{r_t} = s_t + \alpha_N (p_{f,NT,t} - p_{d,NT,t}) + (1 - \alpha_N)/2\; \{[p_{f,TH,t} + p_{f,TF,t}] - [p_{d,TH,t} + p_{d,TF,t}]\}.$$

Chapter 8 – Measuring Persistence

We estimate a regression for $\mathbf{R_t}$ using as explanatory variable $\mathbf{R_{t-1}}$ –i.e., the lagged real exchange rate:

$$\mathbf{R_t} = \boldsymbol{\mu} + \rho\, \mathbf{R_{t-1}} + \varepsilon_t,$$

In finance and economics, this very simple equation describes the behavior over time of a lot of variables. Given this equation, we use ρ as a measure of persistence.

Three cases:

(1) If ρ=0, past $\mathbf{R_t}$'s have no effect on today's $\mathbf{R_t}$. There is no dynamics in $\mathbf{R_t}$; no persistence of shocks to the real exchange rate –i.e., full adjustment to long-run PPP parity:

$$\mathbf{R_t} = \boldsymbol{\mu} + \varepsilon_t,$$

In this case, it is easy to calculate long-run PPP parity –i.e., the mean of $\mathbf{R_t}$ over time:

$$E[\mathbf{R_t}] = \boldsymbol{\mu} \qquad \text{(since } E[\varepsilon_t]=0).$$

Suppose last period there was a shock that deviate $\mathbf{R_t}$ from PPP parity. If ρ=0, last period's shock has no effect on today's $\mathbf{R_t}$ .On average, we are on the long run PPP parity, given by $\boldsymbol{\mu}$:

$$E_t[\mathbf{R_t}] = \boldsymbol{\mu} \qquad \text{(since } E_t[\varepsilon_t]=0.)$$

(2) If 0<ρ<1, there is a gradual adjustment to shocks, depending on ρ. The higher ρ, the slower the adjustment to long run PPP parity. Shocks are persistent. On average:

$$E_t[\mathbf{R_t}] = \boldsymbol{\mu} + \rho\, \mathbf{R_{t-1}}$$

With a little bit of algebra we can calculate the mean of $\mathbf{R_t}$ over time:

$$E[\mathbf{R_t}] = \boldsymbol{\mu} + \rho\, E[\mathbf{R_{t-1}}] \qquad \Rightarrow E[\mathbf{R_t}] = \boldsymbol{\mu}/(1-\rho)$$

(3) If ρ=1, we say that the process generating $\mathbf{R_t}$ contains a *unit root*. We also say $\mathbf{R_t}$ follows a *random walk* process. Shocks never disappear! On average:

$$E_t[\mathbf{R_t}] = \boldsymbol{\mu} + \mathbf{R_{t-1}}$$

In this case, changes in $\mathbf{R_t}$ are predictable: on average, they would be equal to the estimated value $\boldsymbol{\mu}$:

$$E[\mathbf{R_t} - \mathbf{R_{t-1}}] = \boldsymbol{\mu} \qquad \text{(since } E[\varepsilon_t]=0.)$$

But $\mathbf{R_t}$ would, however, not be predictable, even in the long run. Notice that the change each period would be equal to a constant plus an unpredictable random element, ε_t. In the long-run, $\mathbf{R_t}$ will be equal to the sum of the constant $\boldsymbol{\mu}$ each period plus the sum of the ε_t's.

Half-life (H): how long it takes for the initial deviation from $\mathbf{R_t}$ and $\mathbf{R_{t=\infty}}$ (long run PPP parity) to be cut in half. It is estimated by

$$H = -\ln(2)/\ln(\rho)$$

Example: JPY/USD Real exchange rate (Monthly data from 1971-2013)
We estimate a regression for $\mathbf{R_t}$:

$$\mathbf{R_t} = \boldsymbol{\mu} + \rho\, \mathbf{R_{t-1}} + \varepsilon_t,$$

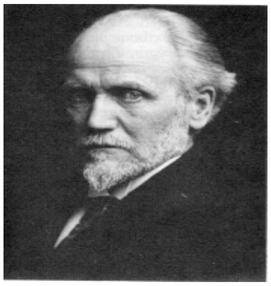
Regression Statistics	
Multiple R	0.986754
R Square	0.973682
Adjusted R Square	0.973631
Standard Error	0.032929
Observations	515

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept ($\boldsymbol{\mu}$)	0.017278	0.006803	2.539632	0.011391
$\mathbf{R_{t-1}}$ (ρ)	0.982179	0.007129	137.7669	0

Calculation of H: H = -ln(2)/ln(.982179) = 38.547 months (or 3.2122 years).

<u>Note</u>: ρ is very high $\Rightarrow$ slow adjustment (high persistence of shocks –i.e., PPP deviations!)
$E[\mathbf{R_t}]$ = long-run PPP parity = $\boldsymbol{\mu}/(1-\rho)$ = 0.017278/(1-.982179) = 0.96953. ¶

The man behind PPP - Karl Gustav Cassel, Sweden (1866 – 1945)



Apart from PPP theory, he produced an 'overconsumption' theory of the trade cycle (1918). He also worked on the German reparations problem. Two of his students, Ohlin and Myrdal, won the Nobel Prize in Economics.